JEE(Main) 2025 | DATE: 29-01-2025 (SHIFT-1) | PAPER-1 | MEMORY BASED

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. **Assertion:** At the peak of mountain, time period of pendulum increases.

   Reason: Time period of pendulum increases with decrease in g.

   (1) Assertion is correct, reason is incorrect

   (2) Assertion is incorrect, reason is correct

   (3) Assertion is incorrect, reason is incorrect

   (4) Assertion is correct, reason is correct

Answer (4)

Sol. $T = 2\pi\sqrt{\frac{l}{g}}$

2. The velocity of a particle moving on a straight line varies with time as $v = At^2 + \frac{Bt}{C+t}$ where A, B, C are constants. Find the dimensions of ABC.

   (1) $L^2 T^{-2}$

   (2) $L^2 T^{-1}$

   (3) $L^2 T^{-3}$

   (4) $L T^{-3}$

Answer (3)

Sol. $[v] = [A][t^2] = \frac{[B][t]}{[C]} = LT^{-1}$

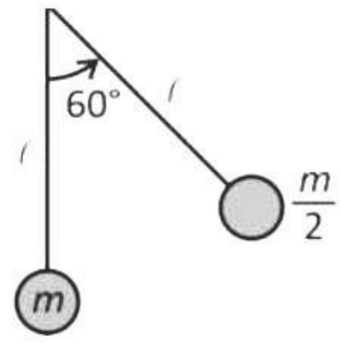
$\Rightarrow [A] = LT^{-3}$

$[B] = LT^{-1}$

$[C] = T$

$[ABC] = L^2 T^{-3}$

3. A pendulum of mass $\frac{m}{2}$ is released from given situation, find speed of another pendulum after collision ($\odot = 1$)

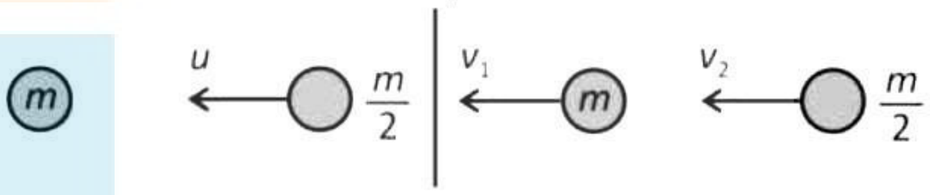


(1) $\sqrt{\frac{3}{2}g\ell}$ (2) $\frac{2}{3}\sqrt{g\ell}$

(3) $\sqrt{\frac{g\ell}{3}}$ (4) $\frac{1}{3}\sqrt{g\ell}$

Answer (2)

Sol. Speed before collision $= \sqrt{2 \cdot g \cdot \frac{\ell}{2}} = \sqrt{g\ell}$



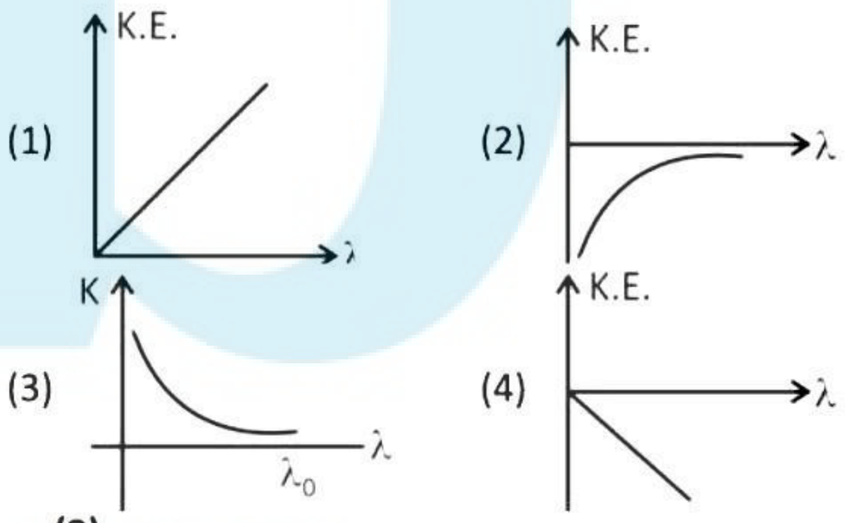
$\frac{m}{2}u = mv_1 + \frac{m}{2}v_2$

$u = 2v_1 + v_2$

$u = v_1 - v_2$

$v_1 = \frac{2u}{3} = \frac{2}{3}\sqrt{g\ell}$

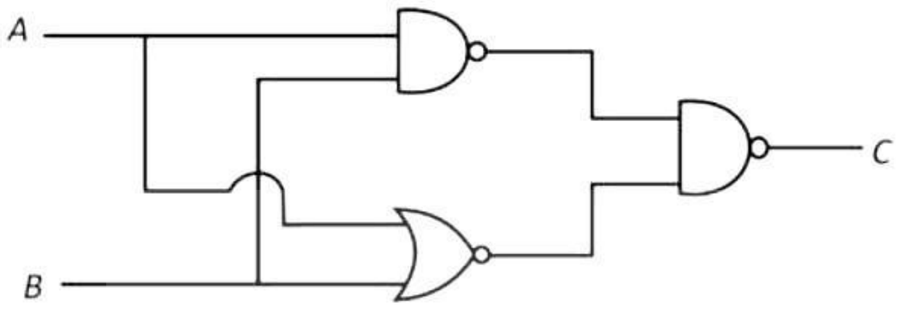
4. The graph between wavelengths (λ) of incident light and kinetic energy (K.E.) of photoelectrons in photoelectric effect is



Answer (3)

Sol. $\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + KE$ $K = \frac{a}{\lambda} - b$

5. Identify the logic gate represented by the circuit shown below.



(1) OR Gate (2) NAND Gate

(3) AND Gate (4) NOR Gate

Answer (1)

Sol. $C = \overline{\left(\overline{AB}\right)\left(\overline{A+B}\right)}$ De Morgan Rule

$= AB + A + B$ $\overline{\overline{X}\,\overline{Y}} = X + Y$

$= A + B$

i.e. OR Gate

6. **Statement-1:** Electromagnetic wave have both energy and momentum.

Statement-2: Rest mass of photon is zero.

(1) Statement-1 is correct, statement-2 is correct

(2) Statement-1 is correct, statement-2 is incorrect

(3) Statement-1 is incorrect, statement-2 is correct

(4) Statement-1 is incorrect, statement-2 is incorrect

Answer (1)

Sol. Because of radiation pressure, EMW exerts force must carry momentum.

According to special relativity theory, no massive particle can attain speed of light.

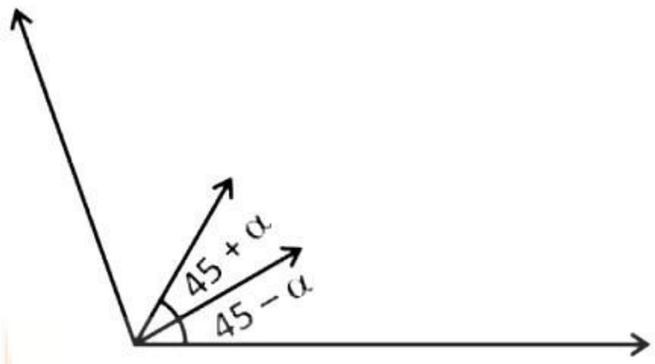
7. Two projectile were launched from same position simultaneously only same speed on of the projectile was launched at angle $(45 - \alpha)^\circ$ and the other at an angle of $(45 + \alpha)^\circ$. Find the ratio of maximum height of the projectile.

(1) $\frac{1-\sin\alpha}{1+\sin\alpha}$ (2) $\frac{1-\sin2\alpha}{1+\sin2\alpha}$

(3) $\frac{1-\tan\alpha}{1+\tan\alpha}$ (4) $\frac{1-\cos\alpha}{1+\cos\alpha}$

Answer (2)

Sol.



$$\frac{2gh_1 = 4^2\sin^2(45-\alpha)}{2gh_2 = 4^2\sin^2(45+\alpha)}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{\left(\frac{\cos\alpha}{\sqrt{2}} - \frac{\sin\alpha}{\sqrt{2}}\right)^2}{\left(\frac{\cos\alpha}{\sqrt{2}} + \frac{\sin\alpha}{\sqrt{2}}\right)^2}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{\cos^2\alpha + \sin^2\alpha - 2\sin\alpha\cos\alpha}{\cos^2\alpha + \sin^2\alpha + 2\sin\alpha\cos\alpha}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{1-\sin2\alpha}{1+\sin2\alpha}$$

8. A river is flowing with speed 9 km/h. Boat is going downstream. Speed of boat in still water is 27 km/h. A person in boat throws a ball upwards with speed 10 m/s. Find range of the ball as seen by an observer at bank of river

(1) 10 m (2) 20 m

(3) 25 m (4) $20\sqrt{3}$ m

Answer (2)

Sol. $T = \frac{2u}{g} = \frac{2\times10}{10} = 2\text{ s}$

$R = (9+27)\frac{5}{18}\times2$

$R = 20$ m

9. Which of two physical quantities have same dimensions?

(1) Angular momentum and Planck's constant

(2) Torque and moment of inertia

(3) Impulse and surface tension

(4) Momentum and work done

Answer (1)

Sol. (1) $\frac{L}{h}=\frac{mvr}{Et}=\frac{mv^2}{E}\equiv M^0L^0T^0$

(2) $\frac{\bar{L}}{I}=\frac{rF\sin\theta}{mr^2}\equiv M^0L^0T^{-2}$

(3) $\frac{I}{s}=\frac{Ft}{F/\ell}\equiv LT$

(4) $\frac{p}{\omega}=\frac{mv}{mv^2}=L^{-1}T$

10. If radius of first Bohr's orbit of H-atom is a_0. Then find the radius of 2nd Bohr's orbit of H-atom.

(1) $8a_0$ (2) $4a_0$

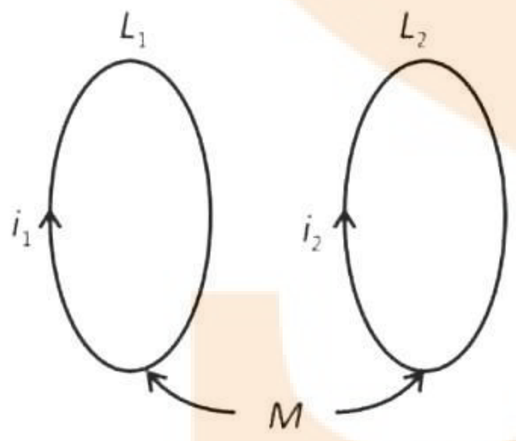
(3) $2a_0$ (4) $6\pi a_0$

Answer (2)

Sol. $a=\frac{a_0n^2}{2}$

So, $a(n=2)=4a_0$

11. Two coils having self-inductance L_1 and L_2 are placed closely such that they have a mutual inductance M. If the carry currents i_1 and i_2 as shown in the figure, then the induced emf in coil 1 is



(1) $-L_1\left(\frac{di_1}{dt}\right)+M\left(\frac{di_2}{dt}\right)$ (2) $-L_1\left(\frac{di_1}{dt}\right)-M\left(\frac{di_2}{dt}\right)$

(3) $-L_1\left(\frac{di_2}{dt}\right)+M\left(\frac{di_1}{dt}\right)$ (4) $-L_1\left(\frac{di_2}{dt}\right)-M\left(\frac{di_1}{dt}\right)$

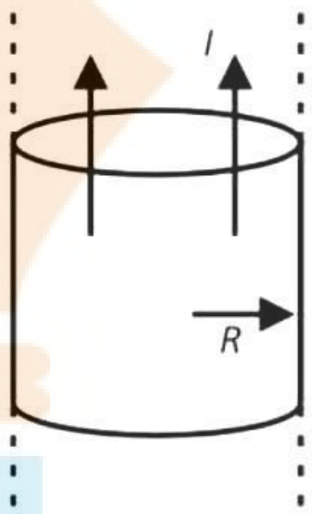
Answer (2)

Sol. $\phi_1=L_1i_1+Mi_2$

$$\frac{-d\phi_1}{dt}=-L_1\left(\frac{di_1}{dt}\right)-M\left(\frac{di_2}{dt}\right)$$

$$\varepsilon_1=-L_1\left(\frac{di_1}{dt}\right)-M\left(\frac{di_2}{dt}\right)$$

12. An infinite solid cylindrical wire of radius R carries a current I uniformly distributed along its area. The distance from the centre where the magnetic field is equal to $\frac{\mu_0 I}{4\pi R}$ is



(1) $\frac{R}{2}$ (2) R

(3) $4R$ (4) 0

Answer (1)

Sol. $B_{inside}=\frac{\mu_0 Ir}{2\pi R^2}$

$$\Rightarrow r=\frac{R}{2}$$

$$B_{outside}=\frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow r=2R$$

13. When ball is kept under sea at depth 2.5 km. Find percentage change in it's volume. If bulk modulus of water is 2×10^9 Pa.

(1) 2% (2) 1.5%

(3) 1.25% (4) 2.75%

Answer (3)

Sol. $\beta = \dfrac{\Delta P}{\dfrac{-\Delta V}{V}} \Rightarrow \dfrac{\Delta V}{V} = \dfrac{\Delta P}{\beta}$

$$= \frac{10^3 \times 10 \times 2500}{2 \times 10^9} \times 100$$

$$= \frac{25}{20}$$

$$= 1.25\%$$

14. Heat given to 0.5 moles of a monoatomic gas at constant pressure is 500 J. Initial temperature of gas was 27°C. Find value of ΔU and ΔT.

(1) 300 J, 48°C (2) 150 J, 24°C

(3) 180 J, 16°C (4) 210 J, 18°C

Answer (1)

Sol. At constant pressure,

$$\Delta Q = nC_p\Delta T$$

$$500 = \frac{n.5}{2}R\Delta T$$

$$\Delta U = nC_v\Delta T = \frac{3}{2}nR\Delta T$$

$$= \frac{3}{2} \times 200$$

$$= 300 \text{ J}$$

$$\Delta T = \frac{200 \times 3}{0.5 \times 25}$$

$$\Delta T = 48$$

15. **Assertion:** A negative potential is required to stop the photoelectron.

Reason : Speed of electron decreases when a negative potential is applied in a photo cell.

(1) Assertion is correct but Reason is false

(2) Assertion is correct and Reason is also correct

(3) Assertion is false but Reason is correct

(4) Assertion is false and Reason is also false

Answer (2)

Sol. Conceptual

16. If electric dipole of dipole moment $\vec{P}$ is placed in electric field $\vec{E}$ with $\vec{P} \,||\, \vec{E}$. It is rotated slightly (and slowly) and released. Find the time period of oscillation of dipole (moment of inertia of dipole is I).

(1) $T = 2\pi\sqrt{\dfrac{I}{PE}}$

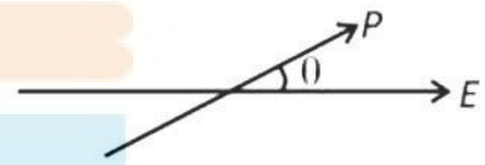
(2) $T = \dfrac{1}{2\pi}\sqrt{\dfrac{PE}{I}}$

(3) $T = 2\pi\sqrt{\dfrac{IE}{P}}$

(4) $T = \dfrac{1}{2\pi}\sqrt{\dfrac{PI}{E}}$

Answer (1)

Sol.



$$T_{(R)} = -(\bar{P})(\bar{E})\sin\theta \approx -|\bar{P}||\bar{E}|\theta$$

$$\alpha = -\omega^2\theta = -\frac{PE}{I} \cdot \theta$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{PE}}$$

17. In adiabatic process of closed system, work done by the gas depends explicitly on

(1) Change in volume

(2) Change in pressure

(3) Change in temperature

(4) Change in number of moles

Answer (3)

Sol. $\Delta\theta = \Delta V + \Delta W \Rightarrow \Delta W = -\Delta V$

$$W = \underbrace{-\frac{\mu R\,\Delta T}{\gamma - 1}}_{\text{Only Change in temperature}} = \underbrace{-\frac{1}{\gamma - 1}(P_2V_2 - P_1V_1)}_{\text{Both on change in pressure and volume}}$$

18. Match the correct option for List-I and List-II, where symbols have usual meanings.

	List-I		List-II
(A)	Electric field inside the spherical shell	(i)	$\frac{\sigma}{2\varepsilon_0}$
(B)	Electric field just outside the spherical shell	(ii)	$\frac{\sigma}{\varepsilon_0}$
(C)	Electric field inside the charged parallel plate capacitor	(iii)	0
(D)	Electric field of infinite charge sheet	(iv)	$\frac{2\sigma}{\varepsilon_0}$

(1) A-(iii), B-(ii), C-(iv), D-(ii)

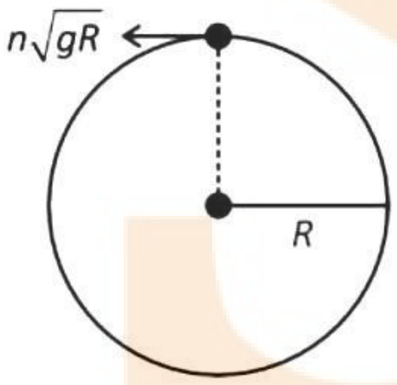
(2) A-(iii), B-(ii), C-(ii), D-(i)

(3) A-(iii), B-(ii), C-(ii), D-(iv)

(4) A-(iv), B-(iii), C-(i), D-(ii)

Answer (2)

19. A particle is able to complete the vertical circular motion with speed $n\sqrt{gR}$ at top-most point. Find the ratio of $\frac{KE_{(Bottom)}}{KE_{(Top)}}$



(1) $\frac{n^2+4}{n}$ (2) $\frac{n}{n^2+4}$

(3) $\frac{n^2+2}{n}$ (4) $\frac{n^2+4}{n^2}$

Answer (4)

Sol. $V_\tau = n\sqrt{gR}$

$$V_{Bottom}^2 = V_\tau^2 + 4gR = n^2gR + 4gR$$

$$\frac{KE_{Bottom}}{KE_{Top}} = \frac{gR(n^2+4)}{gRn^2} = \frac{n^2+4}{n^2}$$

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. In a hydraulic lift, the two sides have areas $A_1 = 25\ cm^2$ and $A_2 = 100\ cm^2$. If a force of 100 N is applied normally on the area A_1, then the force on the area A_2 is _____ N.

Answer (400)

Sol. From Pascal's law

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \text{ or } \frac{100\ N}{25\ cm^2} = \frac{F_2}{100\ cm^2}$$

$$\Rightarrow F_2 = 400\ N$$

22. Find magnitude of component of torque about origin in z-direction when force $\vec{F} = \hat{i} - \hat{j} + \hat{k}$ acts at (1, 1, 1).

Answer (2)

Sol. $\vec{\tau}_2 = \hat{k}(-1,-1) = -2\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & +1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

1. Which of the following is animal starch?
   (1) Glycogen
   (2) Lactose
   (3) Amylopectin
   (4) Amylose

Answer (1)

Sol. Lactose is present in milk.
Amylopectin and amylose are part of starch.
Glycogen is animal starch.

2. **Statement 1 :** Correct order of ionic radius for Mg^{2+}, Na^+, O^{2-}, & F^- is $F^- > O^{2-} > Na^+ > Mg^{2+}$

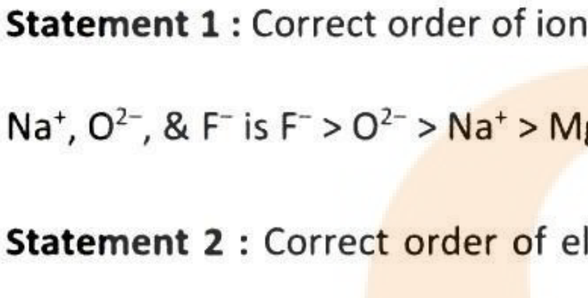
   Statement 2 : Correct order of electron gain enthalpy for 17^{th} group elements follows order Cl > F > Br > I (Magnitude only)

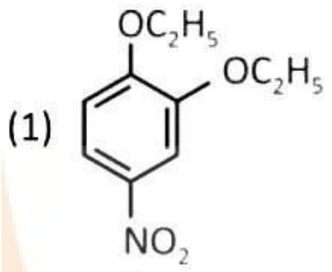
   (1) Statement-1 & Statement-2 are correct
   (2) Statement-1 is correct Statement-2 is incorrect
   (3) Statement-1 & Statement-2 are incorrect
   (4) Statement-1 is incorrect Statement-2 is correct

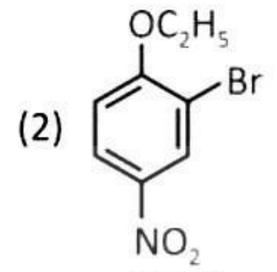
Answer (4)

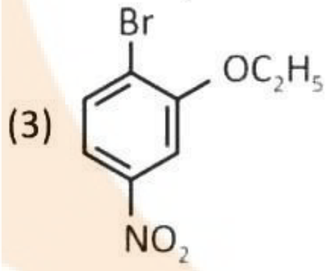
Sol.: Correct order of ionic radius $O^{2-} > F^- > Na^+ > Mg^{2+}$
Correct order for electron gain enthalpy (Magnitude) Cl > F > Br > I

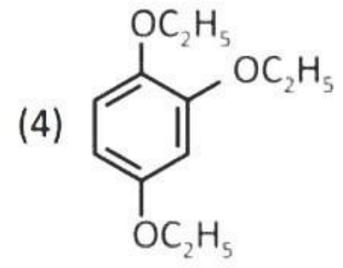
3. Identify the product formed in the following reaction



(1) 

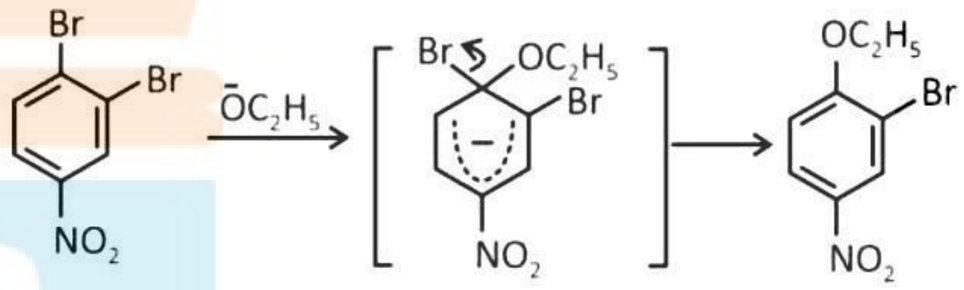
(2) 

(3) 

(4) 

Answer (2)

Sol. Aryl halides having strong electron withdrawing group like NO_2 either at the ortho or para position undergo SNAR reaction easily involving carbanion intermediate



4. Which of the following is steam volatile
   (1) Ortho nitrophenol (2) Para nitrophenol
   (3) Para aminophenol (4) Para nitroaniline

Answer (1)

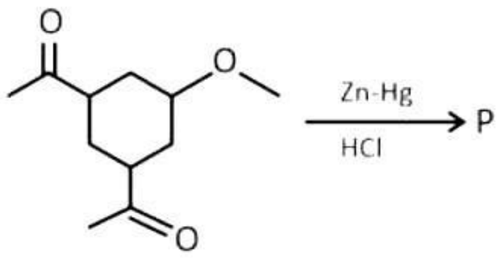
Sol. Ortho nitrophenol is steam volatile due to intramolecular H-bonding It's B.P is less. p-nitrophenol, p-amino phenol, paranitro aniline show intermolecular H-bonding

5. Consider the following complexes
   $[Mn(CN)_6]^{4-}$ $[Fe(CN)_6]^{4-}$ $[Fe(CN)_6]^{3-}$ $[Co(CN)_6]^{3-}$
   (1) (2) (3) (4)
   Correct order of CFSE (Δ) will be
   (1) 3 > 4 > 2 > 1 (2) 4 > 3 > 2 > 1
   (3) 4 > 3 > 1 > 2 (4) 3 > 4 > 1 > 2

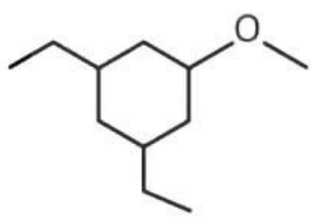
Answer (2)

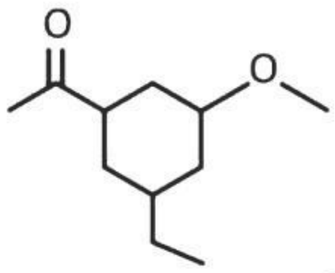
Sol. (1) $[Mn(CN)_6]^{4-}$, Mn^{2+}
(2) $[Fe(CN)_6]^{4-}$, Fe^{2+}
(3) $[Fe(CN)_6]^{3+}$, Fe^{3+}
(4) $[Co(CN)_6]^{3+}$, Co^{3+}
order of CFSE will be 4 > 3 > 2 > 1

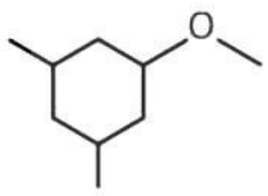
6. Consider the following reaction

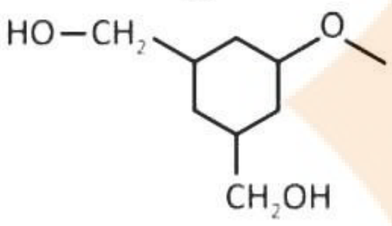


Identify the final product P.

(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. Clemmensen's reduction reagent reduces aldehyde and ketone to alkane.

7. What is the value of van't Hoff Factor for A_2B, if 30% of A_2B is dissociated?

(1) 1.60 (2) 1.30

(3) 1.50 (4) 1.20

Answer (1)

Sol. $\underset{1-\alpha}{A_2B} \rightleftharpoons \underset{2\alpha}{2A^+} + \underset{\alpha}{B^{2-}}$

$i = 1 - \alpha + 2\alpha + \alpha = 1 + 2\alpha$

$\alpha = 0.30$

$i = 1 + 2 \times 0.30 = 1.60$

8. Find the order of the reaction

$A + B \rightarrow F$

if the mechanism of the reaction is as follows:

Step 1 : $A + B \rightarrow D$ (slow)

Step 2 : $D \rightarrow C + E$ (fast)

Step 3 : $C + E \rightarrow F$ (fast)

(1) 1 (2) 3

(3) 2 (4) 4

Answer (3)

Sol. Since the slowest step is considered as rate determining step.

So, here $r = k[A][B]$

Order = 2

9. Match the following **List-I** with **List-II** and choose the correct option

	List-I (Complexes)		**List-II (Hybridisation)**
(A)	$[Co(OX)_3]^{3-}$	(i)	sp^3d^2
(B)	$[FeF_6]^{3-}$	(ii)	d^2sp^3
(C)	$[Ni(CO)_4]$	(iii)	dsp^2
(D)	$[PtCl_4]^{2-}$	(iv)	sp^3

(1) A-(i), B-(ii), C-(iii), D-(iv)

(2) A-(ii), B-(i), C-(iii), D-(iv)

(3) A-(i), B-(ii), C-(iv), D-(iii)

(4) A-(ii), B-(i), C-(iv), D-(iii)

Answer (4)

Sol. : $\left[Co(OX)_3\right]^{3-} \Rightarrow Co^{3+}, (OX)$ act as SFL for Co^{3+}

$\Rightarrow d^6 \Rightarrow t_{2g}^6\, eg^0 \Rightarrow d^2sp^3$ hybridisation

$[FeF_6]^{3-} \Rightarrow Fe^{3+}, F^-$ act as WFL,

$Fe^{3+} \Rightarrow d^5 \Rightarrow t_{2g}^3\, eg^2 \Rightarrow sp^3d^2$ hybridisation.

$[Ni(CO)_4] \Rightarrow$ Ni(0), CO act as SFL

$Ni(0) \Rightarrow s^2d^8 \Rightarrow d^{10} \Rightarrow sp^3$ hybridisation

$[PtCl_4]^{2-} \Rightarrow Pt^{2+} \Rightarrow Cl^-$ act as SFL.

$Pt^{2+} \Rightarrow d^8 \Rightarrow dsp^2$ hybridisation.

10. What is the correct Nernst equation representation for the following cell reaction

$Mg \rightarrow Mg^{2+} + 2e^-$

$Ag^+ + e^- \rightarrow Ag$

(1) $E_{cell} = E^\circ_{cell} - \frac{RT}{2F} \ln \frac{[Mg^{2+}]}{[Ag^+]^2}$

(2) $E_{cell} = E^\circ_{cell} - \frac{RT}{2F} \ln \frac{[Ag^+]^2}{[Mg^{2+}]}$

(3) $E_{cell} = E^{\circ}_{cell} + \frac{RT}{F}\ln\frac{[Mg^{2+}]}{[Ag^{+}]^2}$

(4) $E_{cell} = E^{\circ}_{cell} + \frac{RT}{2F}\ln\frac{[Ag^{+}]^2}{[Mg^{2+}]}$

Answer (1)

Sol. $Mg(s) \rightarrow Mg^{2+}(2q) + 2e^{-}$

$2Ag^{+}(aq) + 2e^{-} \rightarrow 2Ag(s)$

$Mg(s) + 2Ag^{+}(aq) \rightarrow 2Ag(s) + Mg^{2+}(aq)$

$$E_{cell} = E^{\circ}_{cell} - \frac{RT}{2F}\ln\frac{[Mg^{2+}]}{[Ag^{+}]^2}$$

11. The correct order of melting point of d-block elements is :

(1) Fe > Mn (2) Tc > Ru

(3) Os > Re (4) Ta > W

Answer (1)

Sol. Melting point order is Fe > Mn, Ru > Tc, Re > Os, W > Ta

12. Consider the following reaction

$$A_2B(g) \rightleftharpoons A_2(g) + \frac{1}{2}B_2(g)$$

If P is total pressure at equilibrium & K_P is equilibrium constant. Then α in terms of K_P & P is (Assume $\alpha << 1$)

(1) $\sqrt{\frac{K_P}{P}}$ (2) $\sqrt[4]{\frac{K_P}{P}}$

(3) $\sqrt{\frac{2K_P}{P}}$ (4) $\sqrt[3]{\frac{2K_P^2}{P}}$

Answer (4)

Sol. $A_2B(g) \rightleftharpoons A_2(g) + \frac{1}{2}B_2(g)$

t = 0	p_0		
$t = t_{eq}$	$p_0(1-\alpha)$	$p_0\alpha$	$p_0\frac{\alpha}{2}$

$$P = p_0 + p_0\frac{\alpha}{2}$$

$$P = p_0\left(1 + \frac{\alpha}{2}\right) \quad (P \approx p_0)$$

At equilibrium $K_P = \frac{(p_{A_2})\left(p_{B_2}\right)}{(p_{A_2B})} = (\alpha << 1)$

$$k_p = \frac{(p_0\alpha)\left(p_0\frac{\alpha}{2}\right)^{\frac{1}{2}}}{p_0(1-\alpha)} = k_p = \alpha\left(p\frac{\alpha}{2}\right)^{\frac{1}{2}}$$

$$\frac{K_P}{P^{\frac{1}{2}}} = \frac{\alpha^{3/2}}{2^{1/2}}$$

$$\frac{2K_P^2}{P} = \alpha^3$$

$$\boxed{\sqrt[3]{\frac{2K_P^2}{P}} = \alpha}$$

13. $\wedge_m$ is linearly dependent to $\sqrt{c}$ for an electrolyte, then molar conductance for the same electrolyte at infinite dilution shows

(1) Small increase (2) Small decrease

(3) Sharp increase (4) Sharp decrease

Answer (1)

Sol. $\wedge_m$ decreases linearly with $\sqrt{c}$ for strong electrolytes having small –ve slope. It can be extrapolated to $\wedge_m^{\infty}$ as $c \rightarrow 0$.

The molar conductance of the same electrolyte at infinite dilution or as $c \rightarrow 0$ shows small increase.

14. Given ionisation enthalpy of element E(g) is 300 kJ/mol and electron gain enthalpy of A, B, C and D gaseous atoms are –320 kJ/mol, –340 kJ/mol, –200 kJ/mol and –250 kJ/mol, then what will be the correct order of ionic nature of compounds?

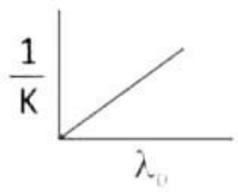
(1) EB > EA > ED > EC (2) EB > EA > EC > ED

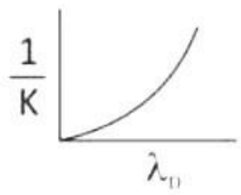
(3) EC > ED > EA > EB (4) EC > ED > EB > EA

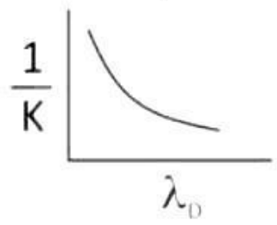
Answer (1)

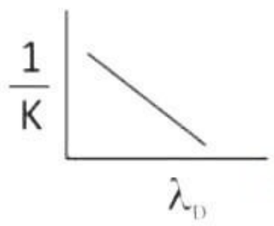
Sol. Since ionic strength depends on IE of electropositive atom; E.G.E. of electronegative element and lattice energy, more the negative value of electron gain enthalpy, more will be ionic nature.

15. Graph between de Broglie wavelength (λ_D) and kinetic energy (K) of an electron is

(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. de Broglie wavelength (λ_D) of an electron of mass (m), moving with velocity (v) is given by

$$\lambda_D = \frac{h}{mv}$$

Where h is planck's constant.

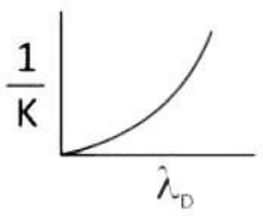
Kinetic energy (K) = $\frac{1}{2}mv^2$

$$mv = \sqrt{2mK}$$

$$\lambda_D = \frac{h}{\sqrt{2mK}}$$

$$\frac{1}{K} = \frac{2m\lambda_D^2}{h^2}$$

Plot of $\frac{1}{K}$ vs λ_D is



16. Which of the following ions is strongest oxidising agent

Given : $E^{\circ}_{Al^{3+}/Al} = -2.7V$

$E^{\circ}_{Cu^{2+}/Cu} = 0.34V$

$E^{\circ}_{Pb^{4+}/Pb^{2+}} = 1.8V$

$E^{\circ}_{Ti^{3+}/Ti^{2+}} = -0.37\,V$

(1) Al^{3+} (2) Cu^{2+}

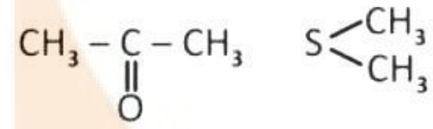
(3) Pb^{4+} (4) Ti^{3+}

Answer (3)

Sol. Reduction potential of $Pb^{4+} \rightarrow Pb^{2+}$ is most positive, Hence Pb^{4+} is strongest oxidising agent.

17. Total number of nucleophiles among the following are

Ph–SH, OH^-, $CH_2 = CH_2$, , H_3O^+,



(1) 5 (2) 6

(3) 7 (4) 4

Answer (2)

Sol. Species having atom containing lone pair available for donation can act as nucleophile

18. Radius of 1st orbit of hydrogen atom is a_0 Å, then find de-Broglie wavelength of 2nd orbit of hydrogen atom.

(1) $4\pi a_0$ (2) $\frac{4}{\pi a_0}$

(3) $8\pi a_0$ (4) $2\pi a_0$

Answer (1)

Sol. $r_n = a_0 \frac{n^2}{Z}$

for n = 1, Z = 1

$r_1 = a_0$

$r_2 = a_0 \frac{4}{1} = 4a_0$

$2\pi r_n = n\lambda$

$\lambda = \frac{2\pi r_2}{2} = \frac{2\pi \times 4a_0}{2} = 4\pi a_0$

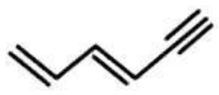
19.

20.

SECTION - B

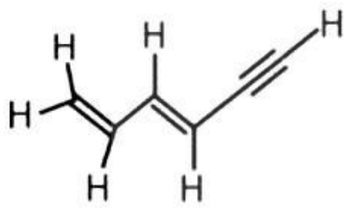
Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Calculate the total number of sigma and π-bonds in the given molecule?



Answer (15)

Sol.



Number of sigma bonds = 11 σ

Number of π-bonds = 4 π

Total = 15

22. Chromite ore + $Na_2CO_3 + O_2 \longrightarrow$ Insoluble product

Calculate the molar mass of insoluble product formed.

(Given : Molar mass of Cr = 52 g/mol, Na = 23 g/mol, Fe = 56 g/mol, 0 = 16 g/mol)

Answer (160)

Sol.

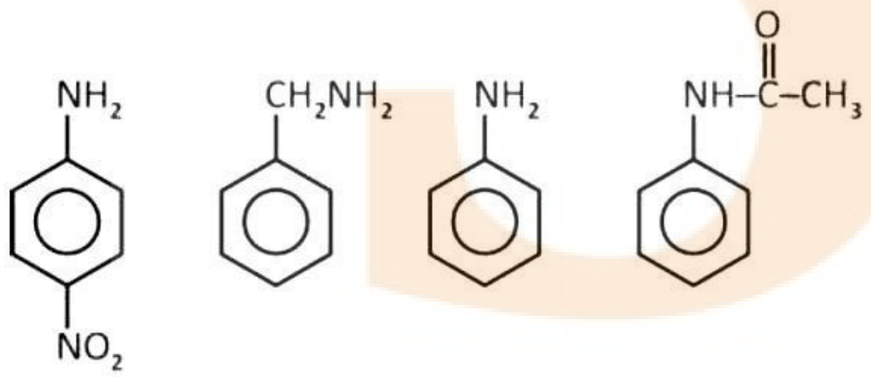
$$\underset{\text{(Chromite ore)}}{4FeCr_2O_4} + 8Na_2CO_3 + 7O_2 \longrightarrow 8Na_2CrO_4 + \underbrace{2Fe_2O_3}_{\text{Insoluble product}} + 8CO_2$$

Molar mass of Fe_2O_3

$\Rightarrow 2(56) + 3(16)$

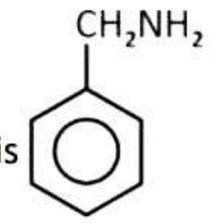
$\Rightarrow 160$

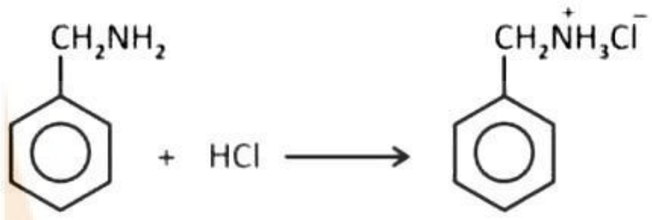
23. Consider the following amines



1 gram of most basic compound reacts with x mg of HCl, calculate value of x.

Answer (341)

Sol. Most basic compound is 



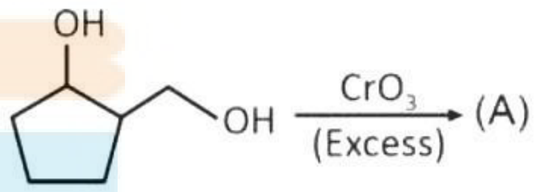
$\frac{1}{107}$ mol $\quad \frac{1}{107}$ mol

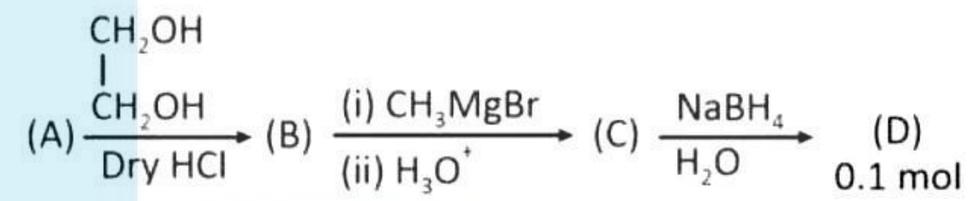
mass of HCl required to react with Benzyl amine

$= \frac{1}{107} \times 36.5\ g$

$= 0.341\ g = 341\ mg$

24. Consider the following reaction

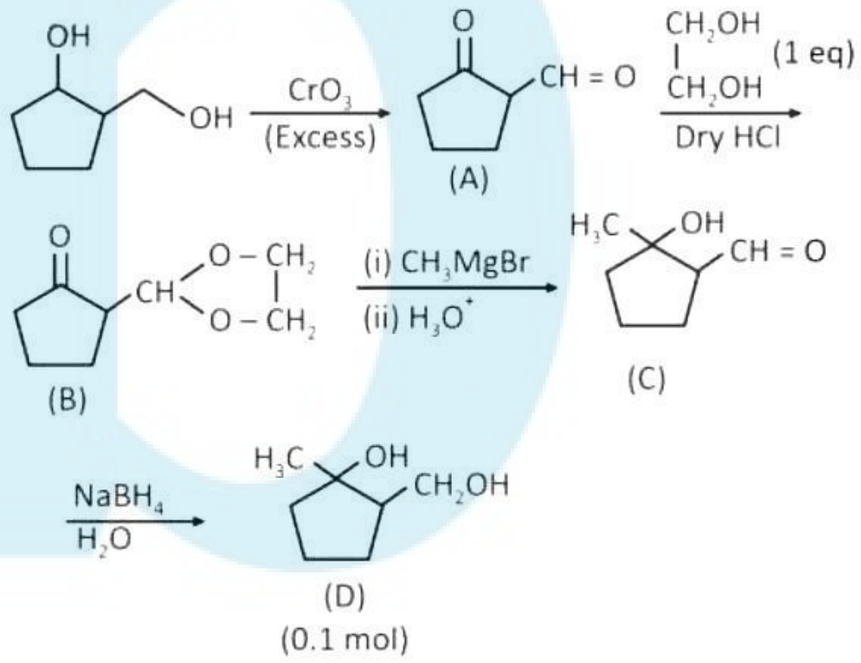




Find the mass of final product(D) formed in g

Answer (13)

Sol.



Molar mass of D = 130 g mol^{-1}

Mass of 0.1 mol of (D) formed = 13g

MATHMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{k^3+6k^2+11k+5}{(k+3)!}$ is equal to

(1) $\frac{5}{3}$ (2) $\frac{8}{3}$

(3) 3 (4) $\frac{7}{3}$

Answer (1)

Sol. $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{k^3+6x^2+11k+6-1}{(k+3)!}$

$=\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{(k+1)(k+2)(k+3)-1}{(k+3)!}$

$=\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{1}{k!}-\dfrac{1}{(k+3)!}$

$=\left(\dfrac{1}{1!}+\dfrac{1}{2!}+\dfrac{1}{3!}+\cdots\infty\right)-\left(\dfrac{1}{4!}+\dfrac{1}{5!}+\dfrac{1}{6!}+\cdots\infty\right)$

$=(e-1)-\left(e-1-\dfrac{1}{1!}-\dfrac{1}{2!}-\dfrac{1}{3!}\right)$

$=1+\dfrac{1}{2}+\dfrac{1}{6}=\dfrac{10}{6}=\dfrac{5}{3}$

2. Sum of first three terms of an AP with integer common difference is 54 and sum of first twenty terms lies between 1600 to 1800, find a_{11}

(1) 108 (2) 90

(3) 111 (4) 115

Answer (2)

Sol. Let AP be a, $a+d$, $a+2d$ ...

$3a+3d=54$

$a+d=18$ ...(i)

$1600<\dfrac{20}{2}[2a+19d]<1800$

$160<2a+19d<180$

$160<18\times2+17d<180$

$\dfrac{124}{7}<d<\dfrac{144}{17}$

$\because d\in\text{Integer}\Rightarrow d=8$

$a+d=18$

$\Rightarrow a=10$

Now $a_{11}=a+10d$

$=10+10\times8$

$=90$

3. Evaluate $I=80\int\limits_0^{\frac{\pi}{2}}\dfrac{\sin x+\cos x}{(9\sin x+16\cos x)}dx$

(1) $\dfrac{80}{327}\left[\dfrac{25\pi}{2}+7\ln\left(\dfrac{9}{16}\right)\right]$

(2) $\dfrac{80}{337}\left[\dfrac{25\pi}{2}-7\ln\left(\dfrac{9}{16}\right)\right]$

(3) $\dfrac{40}{327}\left[\dfrac{25\pi}{2}+7\ln\left(\dfrac{9}{16}\right)\right]$

(4) $\dfrac{40}{327}\left[\dfrac{25\pi}{2}-7\ln\left(\dfrac{9}{16}\right)\right]$

Answer (2)

Sol. $\sin x\cos x=A[9\sin x+16\cos x]+B[9\cos x-16\sin x]$

$=\sin x[9A-16B]+\cos x[16A+9B]$

$\Rightarrow 9A-16B=16A+9B=1$

$\Rightarrow -7A=25B\Rightarrow B=\dfrac{-7A}{25}$

$9A-16\left(\dfrac{-7A}{25}\right)=1\Rightarrow 337A=25,\ B=\dfrac{-7}{337}$

$I=80\int\limits_0^{\frac{\pi}{2}}\dfrac{\frac{25}{337}(9\sin x-16\cos x)\frac{-7}{337}[9\cos x-16\sin x]}{(9\sin x+16\cos x)}dx$

$$I = 80\int_0^{\frac{\pi}{2}} \frac{25}{337}\,dx - 80\int_0^{\frac{\pi}{2}} \frac{\frac{7}{337}\,d(9\sin x + 16\cos x)}{(9\sin x + 16\cos x)}$$

$$I = 80\left(\frac{25x}{337}\right)\Big|_0^{\frac{\pi}{2}} - \frac{80.7}{337}\ln(9\sin x + 16\cos x)\Big|_0^{\frac{\pi}{2}}$$

$$I = \frac{80.25}{337}\left(\frac{\pi}{2}\right) - \frac{80.(7)}{337}\ln\left(\frac{9}{16}\right)$$

4. If R be a relation defined on $(0, \pi/2)$ such that $xRy \Rightarrow \sec^2 x - \tan^2 y = 1$, then the relation R is

(1) Equivalence relation

(2) Reflexive and transitive only

(3) Symmetric and transitive only

(4) Neither reflexive nor transitive

Answer (1)

Sol. $xRy \Rightarrow \sec^2 x - \tan^2 y = 1$

- $xRx \Rightarrow \sec^2 x - \tan^2 x = 1$

$\Rightarrow$ R is reflexive

- $xRy \Rightarrow yRx$

$\Rightarrow$ $\sec^2 x - \tan^2 y = 1$

$\sec^2 y - \tan^2 x = (1 + \tan^2 y) - (\sec^2 x - 1)$

$= 2\sec^2 x + \tan^2 y$

$= 2 - (\sec^2 x - \tan^2 y) = 2 - 1 = 1$

$\Rightarrow$ R is symmetric

- $xRy \Rightarrow yRz$

$\Rightarrow$ $\sec^2 x - \tan^2 y = 1$

$\sec^2 y - \tan^2 z = 1$

Add $\Rightarrow \sec^2 x + \sec^2 y - \tan^2 y - \tan_z^2 = 2$

$\Rightarrow \sec^2 x + (1) - \tan^2 z = 2$

$\Rightarrow \sec^2 x - \tan^2 z = 1$

$\Rightarrow x\,R\,z$

$\Rightarrow$ R is transitive.

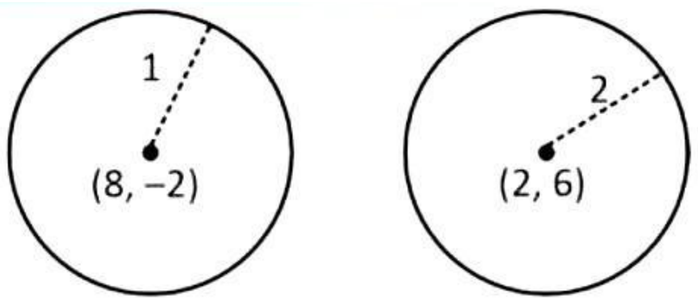
5. If z_1 lies on $|z - 8 + 2i| = 1$ and z_2 lies on $|z - 2 - 6i| = 2$, then $|z_1 - z_2|_{min}$ is

(1) 8 (2) 10

(3) 7 (4) 9

Answer (3)

Sol.



$$|Z_1 - Z_2|_{min} = \sqrt{(8-2)^2 + (2+6)^2} - 3$$

$= \sqrt{36 + 64} - 3$

$= 10 - 3 = 7$

6. If $\cos^{-1}x = \pi + \sin^{-1}x + \sin^{-1}(2x - 1)$, then find the sum of all values of 'x'.

(1) 1 (2) $\frac{1}{2}$

(3) 0 (4) $\frac{3}{2}$

Answer (3)

Sol. $\cos^{-1}x = \pi + \sin^{-1}x + \sin^{-1}(2x - 1)$

Now $-1 \le 2x - 1 \le 1$

$0 \le x \le 1$

$\Rightarrow \quad \pi + \sin^{-1}x + \sin^{1}(2x - 1) \ge \frac{\pi}{2}$

and $\cos^{-1}x$ for $x \in [0, 1]$ always lies in $\left[0, \frac{\pi}{2}\right]$

$\Rightarrow$ LHS = RHS = $\frac{\pi}{2}$

$\Rightarrow \quad \cos^{-1}x = \frac{\pi}{2} \Rightarrow \boxed{x = 0}$

Hence only $x = 0$ is the possible solution.

Sun of all solution = 0.

7. If $\begin{vmatrix} \sin^2 x & 1+\cos^2 x & \sin 4x \\ 1+\sin^2 x & \cos^2 x & \sin 4x \\ \sin^2 x & \cos^2 x & 1+\sin 4x \end{vmatrix} = L$

and $L_{min} = m$ and $L_{max} = M$, then $|M^4 - m^4|$ is

(1) 79 (2) 78

(3) 80 (4) 76

Answer (3)

Sol. $\begin{vmatrix} \sin^2 x & 1+\cos^2 x & \sin 4x \\ 1+\sin^2 x & \cos^2 x & \sin 4x \\ \sin^2 x & \cos^2 x & 1+\sin 4x \end{vmatrix} = -\sin(4x) - 2 = L$

$L_{min} = -3 = m \quad L_{max} = -1 = M$

$\therefore \quad m^4 - M^4 = 81 - 1 = 80$

8. If α, β are real numbers such that $\sec^2(\tan^{-1}\alpha) + \operatorname{cosec}^2(\cot^{-1}\beta) = 36$ and $\alpha + \beta = 8$, where $\alpha > \beta$, then $(\alpha^3 + \beta^3)$ is equal to

(1) 146 (2) 152

(3) 148 (4) 150

Answer (2)

Sol. Let $A = \tan^{-1}(\alpha)$, $B = \cot^{-1}(\beta)$

$\Rightarrow \alpha = \tan A, \beta = \cot B$

$\Rightarrow \tan A + \cot B = 8$

$\sec^2(A) + \operatorname{cosec}^2(B) = 36$

$\Rightarrow 1 + \tan^2 A + 1 + \cot^2 B = 36$

$\Rightarrow \tan^2 A + \cot^2 B = 34$

$\Rightarrow (\tan A + \cot B)^2 = 64 = 34 + 2\tan A.\cot B$

$\Rightarrow \tan A.\cot B = 15$

$\Rightarrow x^2 - 8x + 25 = 0$ has roots $\tan A, \cot B$

$\Rightarrow \tan A = 5, \cot B = 3$

As $\alpha > \beta$

$\Rightarrow \alpha^3 > \beta^3 = (\tan A)^3 + (\cot B)^3 = 5^3 + 3^3 = 125 + 27 = 152$

9. How many 6 letter words can be formed using the word MATHS such that any letter can be used maximum two times.

(1) 6400 (2) 8100

(3) 10000 (4) 9824

Answer (2)

Sol. MATHS has only 5 letters, so in a 6-letter word at least one letter has to repeat.

Let's make cases:

(i) **Case-I:** Exactly one letter is repeated twice.

$^5C_1 \cdot \frac{6!}{2!}$ MM ATHS

(ii) **Case-II:** Exactly two letters are repeated twice.

$^5C_2 \cdot {}^3C_2 \frac{6!}{2!2!}$ MM AA THS

(iii) **Case-III:** Exactly 3 letters are repeated twice

$^5C_3 \frac{6!}{2!2!2!}$

$\therefore$ Required words = 8100

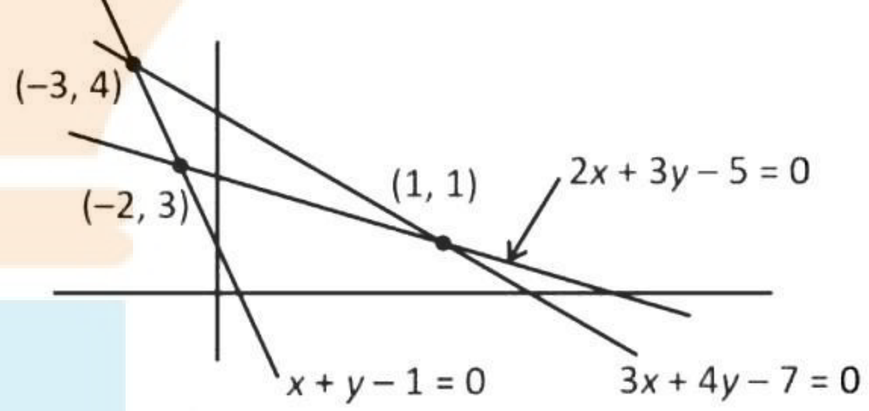
10. A triangle is formed by three lines $2x + 3y - 5 = 0$, $x + y - 1 = 0$, $3x + 4y - 7 = 0$. Let (h, k) be the image of the centroid of ΔABC in the line $2x + 4y - 7 = 0$ then $h^2 + k^2 + hk$ is

(1) $\frac{903}{225}$ (2) $\frac{223}{225}$

(3) $\frac{100}{23}$ (4) $\frac{10006}{225}$

Answer (1)

Sol.



Centroid $\left(\frac{-3+1-2}{3}, \frac{4+3+1}{3}\right)\left(\frac{-4}{3}, \frac{8}{3}\right)$

Image of $\left(\frac{-4}{3}, \frac{8}{3}\right)$ in $2x + 4y - 7 = 0$ is (h,k)

$$\frac{h + \frac{4}{3}}{2} = \frac{k - \frac{8}{3}}{4} = -2\left[\frac{\frac{-8}{3} + \frac{32}{3} - 7}{4+16}\right]$$

$$\frac{h + \frac{4}{3}}{2} = \frac{k - \frac{8}{3}}{4} = \frac{-1}{10}$$

$$h = -\frac{1}{5} - \frac{4}{3} = -\frac{23}{15}$$

$$k = -\frac{2}{5} + \frac{8}{3} = -\frac{35}{15}$$

$$h^2 + k^2 + hk = \left(\frac{-23}{15}\right)^2 + \left(\frac{35}{15}\right)^2 - \frac{23}{15} \times \frac{34}{15}$$

$$= \frac{903}{225}$$

11. If two lines $L_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2}$;

$L_2 : \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z}{1}$. Let the line L_3 passes through the point (α, β, γ) such that L_3 is perpendicular to L_1 to L_2 and L_3 intersects L_1. Then $|5\alpha - 11\beta - 8\gamma|$ is equal to

(1) 18

(2) 25

(3) 16

(4) 20

Answer (2)

Sol. Let the L_3 be

$\frac{x-\alpha}{a} = \frac{y-\beta}{b} = \frac{z-\gamma}{c}$, $(a\hat{i} + \hat{b} + c\hat{k})$ is parallel to

$(\hat{i} - \hat{j} + 2\hat{k}) \times (-\hat{i} + 2\hat{j} + \hat{k})$

$(a, b, c) \equiv (5, 3, 1)$

$\Rightarrow \frac{x-\alpha}{5} = \frac{y-\beta}{3} = \frac{z-\gamma}{-1}$

$\Rightarrow$ Let the point of intersection be P.

$\Rightarrow 5\lambda + \alpha = P + 1, 3\lambda + \beta = P + 2, -\lambda + \gamma = 2P + 1$

$\Rightarrow \alpha = (P + 1 - 5\lambda), \beta = (-P + 2 - 3\lambda), \gamma = (2P + 1 + \lambda)$

$\Rightarrow |5\alpha - 11\beta - 8\gamma| = |-25| = 25$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The minimum value of n for which the number of integer terms in the binomial expansion $\left(7^{\frac{1}{3}} + 11^{\frac{1}{12}}\right)^n$ is 183, is

Answer (2184)

Sol. $T_{k+1} = {}^nC_K \cdot \left(11^{\frac{1}{12}}\right)^k \cdot 7^{\frac{1}{3}(n-k)}$

$12|k$ and $3|(n-k) \Rightarrow 3|n$

For integer terms.

$\Rightarrow$ Multiples of 12 for k would work.

$\Rightarrow k = 0, 12, 24, \ldots$

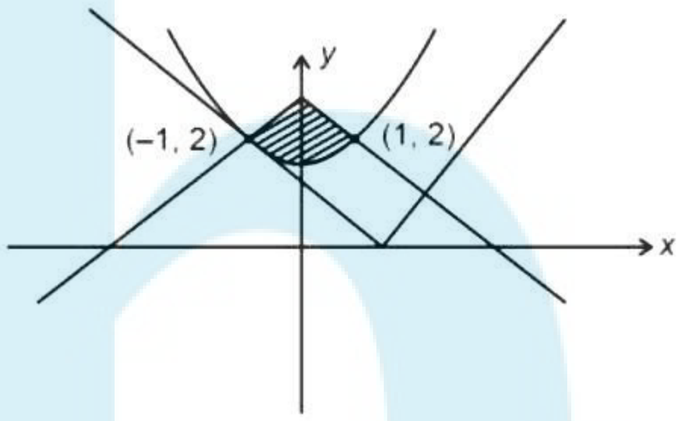
$\Rightarrow k_{max} = 12 \times 182 = 2184$

$\Rightarrow$ Minimum value of n will be 2184 as $3|2184$.

22. Area enclosed by $y \geq |x-1|, y + |x| \leq 3, x^2 \leq 2y - 3$ is A, then $6A$ is (in sq. units)

Answer (10)

Sol.



$$\text{Area} = 2\left[\int_0^1 (3-x) - \left(\frac{x^2+3}{2}\right)dx\right]$$

$$= 2\left[3x - \frac{x^2}{2} - \frac{1}{2}\left[\frac{x^3}{3} + 3x\right]\right]_0^1$$

$$= 2\left(3 - \frac{1}{2} - \frac{1}{2}\left[\frac{1}{3} + 3\right]\right)$$

$$= 2\left(\frac{5}{6} - \frac{1}{6} - \frac{3}{2}\right) = 2\left(\frac{5}{6}\right) = A$$

$6A = 10$

23. Number of 7 digit numbers made with the digits 1, 2, 3 such that sum of the digits is 11 is equal to

Answer (161)

Sol. **Case-I :** 3 2 2 1 1 1 1

$$n_1 = \frac{7!}{4!2!} = 105$$

Case II: 2 2 2 2 1 1 1

$$\Rightarrow n_2 = \frac{7!}{4!3!} = 35$$

Case III : 3 3 1 1 1 1 1

$$\Rightarrow n_3 = \frac{7!}{5!2!} = 21$$

Total numbers $n_1 + n_2 + n_3$

$= 105 + 35 + 21$

$= 161$

24. The minimum value of p such that

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{p}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2}{x^2}\right]+\ldots+\left[\frac{9}{x^2}\right]\right)\geq 1,$$

is equal to (where [.] represents greatest integer function)

Answer (24)

Sol. Since $x^2\left[\frac{r^2}{x^2}\right] = x^2\left(\frac{r^2}{x^2}-\left\{\frac{r^2}{x^2}\right\}\right)$

$$= r^2 - x^2\left\{\frac{r^2}{x^2}\right\}$$

$$\lim_{x\to 0^+} x^2\left[\frac{r^2}{x^2}\right] = \lim_{x\to 0^+} r^2 - x^2\left\{\frac{r^2}{x^2}\right\} = r^2$$

Also,

$$\lim_{x\to 0^+} x\left[\frac{k}{x}\right] = \lim_{x\to 0^+} x\left(\frac{k}{x}-\left\{\frac{k}{x}\right\}\right) = \lim_{x\to 0^+} k - x\left\{\frac{k}{x}\right\}$$

$$= k$$

$$\Rightarrow \lim_{x\to 0^+}\left(\sum_{k=1}^{p} x\left[\frac{k}{x}\right]-\sum_{k=1}^{9} x^2\left[\frac{k^2}{x^2}\right]\right)$$

$$= \sum_{k=1}^{p}\lim_{x\to 0^+} x\left[\frac{k}{x}\right]-\sum_{k=1}^{9}\lim_{x\to 0^+} x^2\left[\frac{k^2}{x^2}\right]$$

$$== \sum_{k=1}^{p} k - \sum_{k=1}^{9} k^2$$

$$= \frac{p(p+1)}{2} - \frac{(9)(10)(19)}{6} \geq 1$$

$$\Rightarrow \quad = \frac{p(p+1)}{2} - 285 \geq 1$$

$$\Rightarrow \quad p(p+1) \geq 2.286$$

$$\Rightarrow \quad p(p+1) \geq 572$$

Clearly $p = 23$ doesn't satisfy

$\Rightarrow$ Minimum value is $p = 24$, as $24^2 = 576 > 572$

25. Two parabolas having common focus at (4, 3) intersect at points A and B. Find the value of $(AB)^2$, given that directrices of these parabolas are along X-axis and Y-axis respectively.

Answer (192)

Sol. Equation of parabolas:

$(x-y)^2 + (y-3)^2 = x^2$

$(x-y)^2 + (y-3)^2 = y^2$

Let they intersect at (x_1, y_1) and (x_2, y_2)

$\therefore\ x_1^2 = y_1^2 \quad \Rightarrow\ x_1 = y_1 \quad (x_1 > 0, y_1 > 0)$

$\therefore\ (x_1-4)^2 + (x_1-3)^2 = x_1^2$

$$\Rightarrow \quad x_1^2 - 14x_1 + 25 = 0$$

$x_1 + x_2 = 14,\ x_1 \cdot x_2 = 25$

$(AB)^2 = \left(\sqrt{(x_1-x_2)+(y_1-y_2)}\right)^2$

$= 2(x_1-x_2)^2$

$= 2((x_1+x_2)^2 - 4x_1x_2)$

$= 2(196-100)$

$= 192$

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